Lab 6: Fourier Transform
Fall 2021
EECS 360: Signal and System Analysis

We already know about Fourier series that acts on periodic function. Today we are going to learn Fourier transform that acts on non – periodic function. **We mostly use FFT (Fast Fourier Transform) and DFT (Discrete Fourier Transform) in MATLAB to perform Fourier transform on data.** However, we can uses inbuilt MATLAB function called "fourier" to perform Fourier transform. But we don't use this inbuilt that much in signal processing. We will learn FFT and DFT in our next two lab. **Today, we will learn how to do Fourier Transform by hand, Fourier Transform of some important signals, properties of Fourier Transform and finally will write some MATLAB code using inbuilt "fourier" command.**

Mathematical Definition of Fourier Transform

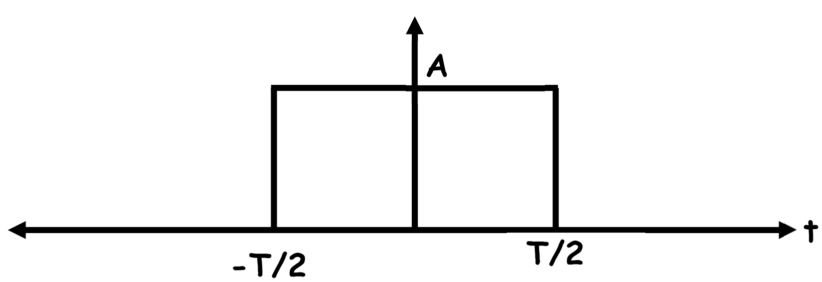
Let's think we have a time domain non – periodic function $f(t)$. The Fourier transform of this function is:

$$F(\omega) = \int_{-\infty}^{\infty} f(t) e^{-j\omega t} dt$$

Inverse Fourier Transform is, $F(t) = \dfrac{1}{2\pi} \int_{-\infty}^{\infty} F(\omega) e^{j\omega t} d\omega$

Let's do some example:

1. Fourier Transform of Rectangular Pulse

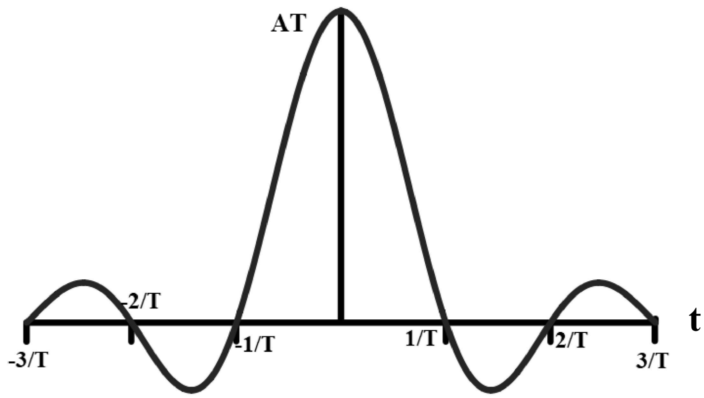


Here, we can define the function as: $f(t) = \begin{cases} A; & -\dfrac{T}{2} \le t \le \dfrac{T}{2} \\ 0; & otherwise \end{cases}$

Using the Fourier Transform Formula, we can calculate the Fourier transform:

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt = \int_{-T/2}^{T/2} Ae^{-j\omega t}dt = A \times \frac{1}{-j\omega}\left[e^{-j\omega t}\right]_{-T/2}^{T/2} = A \times (-) \times \frac{1}{j\omega} \times (e^{-j\omega\frac{T}{2}} - e^{j\omega\frac{T}{2}})$$

$$F(\omega) == A \times \frac{1}{j\omega} \times (e^{j\omega\frac{T}{2}} - e^{-j\omega\frac{T}{2}}) = AT \times \frac{(e^{j\omega\frac{T}{2}} - e^{-j\omega\frac{T}{2}})}{\frac{\omega T}{2} \times 2j} = AT \times \frac{\sin(\frac{\omega T}{2})}{\frac{\omega T}{2}} = AT \times \mathrm{sinc}(\frac{\omega T}{2})$$



If we draw it in ω scale, each point in time scale will be multiplied by 2π

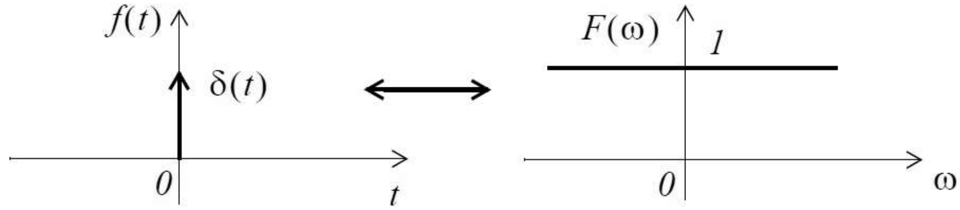
We can easily do Fourier transform of a Rectangular Pulse just remembering the following points:

1. Fourier Transform of a rectangular pulse is always a sinc function.
2. The amplitude of the since will be $A \times T$ where T is the width of the pulse.
3. Zero crossing of the sinc pulse in X – axis will always be at integer multiple of $1/T$

2. Fourier Transform of a Single Impulse

Here, we can define the Impulse function as: $f(t) = \begin{cases} 1; & t = 0 \\ 0; & otherwise \end{cases}$

$$F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt = \int_{-\infty}^{\infty} e^{-j\omega t}dt = e^{-j\omega \times 0} = 1$$



Fourier Transform of a Shifted Delta Function is: $\delta(t - t_0) = e^{-j\omega t_0}$

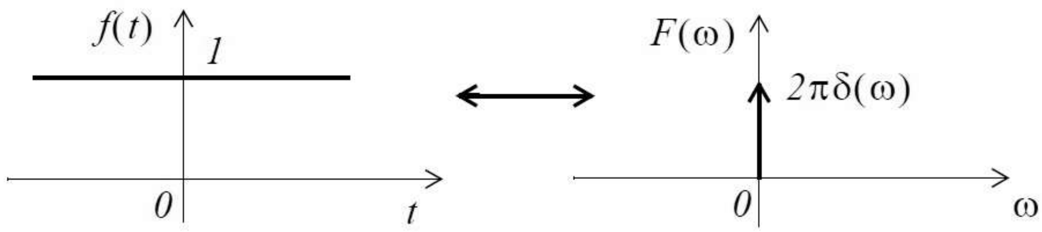
3. Fourier Transform of a Constant Amplitude

Let's say, $f(t) = 1; -\infty < t < \infty$

The Fourier transform will be, $F(\omega) = \int_{-\infty}^{\infty} f(t)e^{-j\omega t}dt = \int_{-\infty}^{\infty} e^{-j\omega t}dt = 2\pi\delta(\omega)$

We can prove this by using inverse Fourier transform formula.

$$F(t) = \frac{1}{2\pi}\int_{-\infty}^{\infty} F(\omega)e^{j\omega t}d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} 2\pi\delta(\omega)e^{j\omega t}d\omega = \left|e^{j\omega t}\right|_{\omega=0} = 1$$



Similar way we can show, $f(e^{j\omega_0 t}) = 2\pi\delta(\omega - \omega_0)$

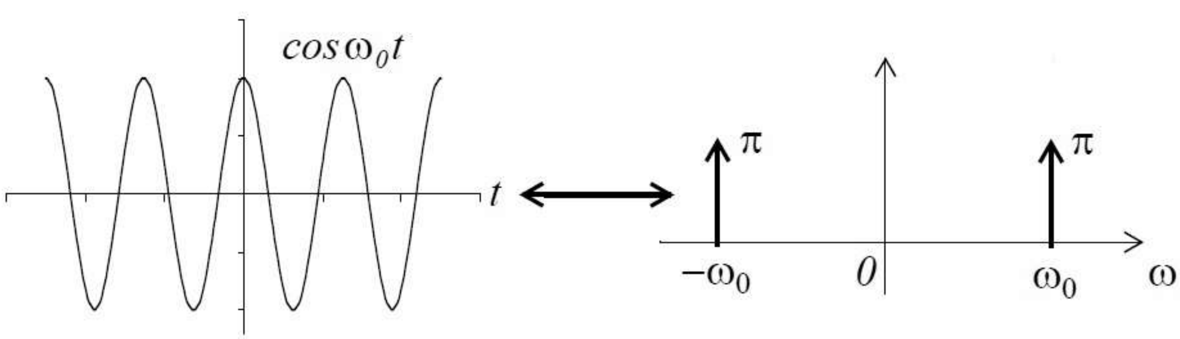
4. Fourier Transform of a Cos function

We can write cos function as, $f(t) = \cos(\omega_0 t) = \frac{1}{2}(e^{j\omega_0 t} + e^{-j\omega_0 t})$

Fourier Transform follows linearity property. So, we can think the cos function as summation of $\frac{1}{2}e^{j\omega_0 t}$ & $\frac{1}{2}e^{-j\omega_0 t}$. Using the fact, $f(e^{j\omega_0 t}) = 2\pi\delta(\omega - \omega_0)$ we can write,

$$f(\frac{1}{2}e^{j\omega_0 t}) = \frac{1}{2} \times 2\pi\delta(\omega - \omega_0) = \pi\delta(\omega - \omega_0)$$
$$f(\frac{1}{2}e^{-j\omega_0 t}) = \frac{1}{2} \times 2\pi\delta(\omega + \omega_0) = \pi\delta(\omega + \omega_0)$$

$$\therefore f[\cos(\omega_0 t)] = \pi\delta(\omega - \omega_0) + \pi\delta(\omega + \omega_0)$$



5. Important Properties of Fourier Transform

(a) Linearity (Superposition)

$$if, x_1(t) \leftrightarrow X_1(\omega) \,\&\, x_2(t) \leftrightarrow X_2(\omega); then$$
$$x_1(t) + x_2(t) \leftrightarrow X_1(\omega) + X_2(\omega)$$

(b) Time Shifting

$$if, x(t) \leftrightarrow X(\omega); then$$
$$x(t - t_0) \leftrightarrow X(\omega) \times e^{-j\omega t_0}$$

N.B: Therefore, the amplitude spectrum of the time shifted signal is the same as the amplitude spectrum of the original signal, and the phase spectrum of the time-shifted signal is the sum of the phase spectrum of the original signal and a linear phase term.

(c) Time Scaling

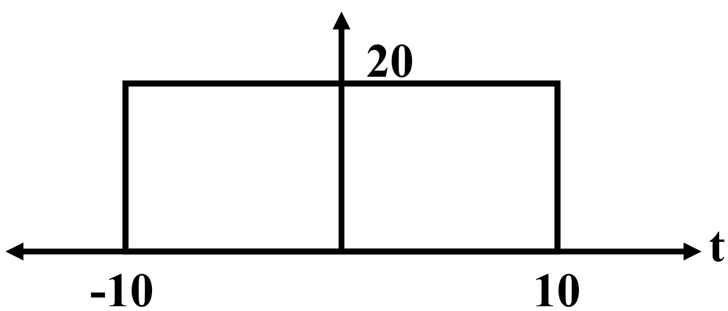
$$if, x(t) \leftrightarrow X(\omega), then$$
$$x(at) \leftrightarrow \frac{1}{|a|} X(\frac{\omega}{a})$$

(d) Frequency Shifting

$$if, x(t) \leftrightarrow X(\omega); then$$
$$x(t) \times e^{-j\omega_0 t} \leftrightarrow X(\omega - \omega_0)$$

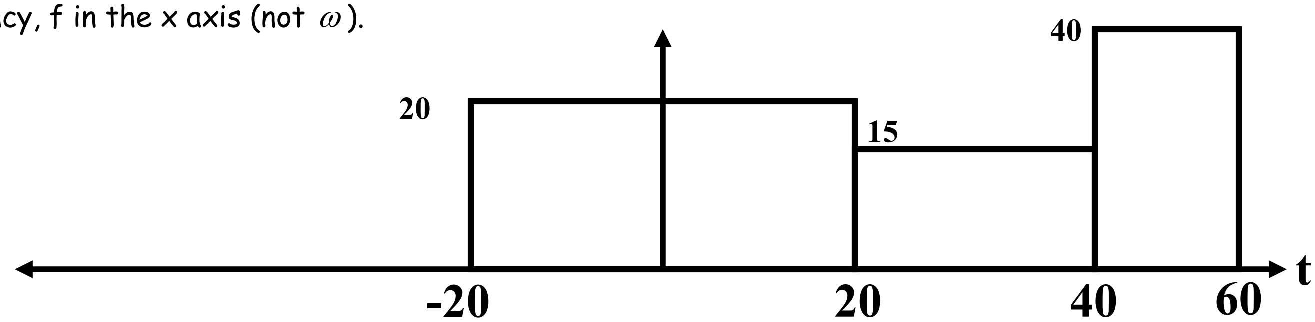
LAB ASSIGNMENTS:

1.



Find the Fourier transform of the above signal and plot the amplitude and phase plot of the signal. Put frequency, f in the x axis (not ω).

2.



Find the Fourier transform of the above signal and plot the amplitude and phase plot of the signal. Put frequency, f in the x axis (not ω).